$+(1n^{n+1}+2^{n+1}+3^{n+1}+4^{n+1}+\ldots\ldots+n^{n+1})a_n.$

Transposing the last quantity in parenthesis, reducing, and replacing the values of a_1, a_2, a_3, we get,

$$(1+2+3+\ldots\ldots+n)+\frac{n}{2\,!}(1^2+2^2+3^2+\ldots\ldots+n^2)+\frac{n(n-1)}{3\,!}(1^3+2^3+3^3+\ldots n^3)$$

$$+\ldots\ldots+\frac{n(n-1)}{3\,!}(1^{n-2}+2^{n-2}+3^{n-2}+\ldots\ldots+n^{n-2})+\frac{n}{2\,!}(1^{n-1}+2^{n-1}+3^{n-1}$$

$$+\ldots\ldots+n^{n-1})+(1^n+2^n+3^n+\ldots\ldots+n^n)=(n+1)^n-1.$$

When $n=2$, we get $(1+2)+(1^2+2^2)=3^2-1=8$.

When $n=3$, $(1+2+3)+\frac{3}{2}(1^2+2^2+3^2)+(1^3+2^3+3^3)=4^3-1=63$.

When $n=4$, $(1+2+3+4)+2(1^2+2^2+3^2+4^2)+2(1^3+2^3+3^3+4^3)$ $+(1^4+2^4+3^4+4^4)=5^4-1=624$.

When $n=5$, $(1+2+3+4+5)+\frac{5}{2}(1^2+2^2+3^2+4^2+5^2)+\frac{10}{3}(1^3+2^3+3^3$ $+4^3+5^3)+\frac{5}{2}(1^4+2^4+3^4+4^4+5^4)+(1^5+2^5+3^5+4^5+5^5)=6^5-1=7775$.

When $n=6$, $(1+2+3+4+5+6)+3(1^2+2^2+3^2+4^2+5^2+6^2)+5(1^3+2^3+3^3$ $+4^3+5^3+6^3)+5(1^4+2^4+3^4+4^4+5^4+6^4)+3(1^5+2^5+3^5+4^5+5^5+6^5)+(1^6+2^6+3^6$ $+4^6+5^6+6^6)=7^6-1=117648$.

Also solved by *ELMER SCHUYLER.*

102. **Proposed by J. MARCUS BOORMAN, Woodmere, N. Y.**

Solve $2x+\sqrt{[x^2-7]}=5$.

I. **Solution by COOPER D. SCHMITT. A. M., University of Tennessee, Knoxville, Tenn.; M. A. GRUBER. A. M., Washington, D. C.; J. SCHEFFER, A. M., Hagerstown, Md ; and G. B. M. ZERR. A. M., Ph. D., Chester High School, Chester, Pa.**

Transposing, $\sqrt{(x^2-7)}=5-2x$.

Squaring, $x^2-7=25-20x+4x^2$; whence $3x^2-20x+32=0$.

Solving, $x=4$ or $\frac{8}{3}$.

Neither of these satisfies the original equation, but by writing it thus, $2x-\sqrt{(x^2-7)}=5$, both values will satisfy it.

II. **Solution by H. C. WHITAKER. M. E., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.**

Denote $2x-5$ by y; then the given equation reduces to

$$+\sqrt{(y^2+10y-3)}=-2y.$$

Divide by y, $\quad +\sqrt{(1+10/y-3/y^2)}=-2\ldots\ldots(1).$

But this equation is absurd, since it makes a positive square root equal to a negative number.

Assume the symbolism $+\sqrt{j}=-1$, j *being an impossible quantity, the root of the equation* $+\sqrt{x}+1=0$.

Square (1), $1+10/y-3/y^2=4j$, $y^2(4j-1)-10y+3=0$, from which

$$x=\frac{10j\pm\sqrt{(7-3j)}}{4j-1}.$$

III. Solution by the PROPOSER.

Transpose and square.

$\therefore\ 4x^2-20x+25=x^2-7\ldots\ldots(B)$, an obvious quadratic.

Apply its roots, 4 and $\frac{8}{3}$, to the given (A); hence $2(4)+[-3]=8-3=5$; $\ldots\ldots=2x+\{-[\sqrt{(16-7)}]\}\ldots\ldots(C)$; and

$$2(\tfrac{8}{3})+(-\tfrac{1}{3})=5\tfrac{1}{3}-\tfrac{1}{3}=5\ldots\ldots=2x+\{-[\sqrt{(\tfrac{64}{9}-\tfrac{63}{9})}]\}\ldots\ldots(D);$$

satisfy it. Could extracting $\sqrt{(x^2-7)}$ *positive* here *also* yield roots, then (A)'s dominant *quadratic* (B) is *bi*-quadratic, which is absurd.

Also solved by *P. S. BERG* and *CHAS. C. CROSS*.

GEOMETRY.

127. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the plane through the extremities, (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), of conjugate diameters of the ellipsoid,

$$\frac{x^2}{y^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \text{ is } \frac{x_1+x_2+x_3}{a^2}x+\frac{y_1+y_2+y_3}{b^2}y+\frac{z_1+z_2+z_3}{c^2}z=1.$$

Solution by the PROPOSER.

If $lx+my+nz=p$ ....(1) be the required plane, we should have

$$lx_1+my_1+nz_1=p\ldots\ldots(2),$$
$$lx_2+my_2+nz_2=p\ldots\ldots(3),$$
$$lx_3+my_3+nz_3=p\ldots\ldots(4).$$

Solving these for l/p, m/p, n/p, we have

$$l/p=\begin{vmatrix}1, & y_1, & z_1\\ 1, & y_2, & z_2\\ 1, & y_3, & z_3\end{vmatrix}\div\begin{vmatrix}x_1, & y_1, & z_1\\ x_2, & y_2, & z_2\\ x_3, & y_3, & z_3\end{vmatrix}\ldots\ldots(5).$$

$m/p=$etc., $n/p=$etc., ...(6).

Reducing (5), making use of